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Buckling Behaviour of Linearly Tapered Steel Columns using FEM

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Abstract:

This paper presents the buckling behaviour of tapered columns which are widely used in structural engineering applications such as bridges and towers. Buckling of thinned structural members subjected to compression forces have been studied for long time, and the buckling loads for different boundary conditions have obtained mathematically by solving the differential equations for each case considering the moment of inertia is constant through the entire section, while in case of non-uniform cross section, tapered sections are more complex compared to prismatic sections. In order to solve these problems, numerical methods become the suitable tools, such as finite integration or finite element methods, or any other methods. Finite element analysis is presented in this paper in order to investigate the behaviour of the buckling columns by including initial imperfections. The columns are lineally tapered W-sections with variable moment of inertia about strong axis, variable moment of inertia in weak axis or both axis were not considered. Abaqus-2014 software was used to analyse the columns and the buckling loads were obtained for different taper ratios. FEM analysis consists of four models for each selected W-section in order to investigate the effects of taper ratio on the buckling load. Based on this investigation it is concluded that taper ratio significantly affects the critical buckling capacity for the same material properties and length. The exact effect depends on the boundary conditions and taper profile. The comparison of the buckling moment between prismatic section with the tapered section ranges from 4%-12% for W8×18 sections and 8%-20% for W14×30 sections.

Keywords: lateral-torsional buckling; tapered steel member; Taper ratio; finite element analysis; geometric imperfection

1. Introduction

Structural steel members with non-prismatic cross-sections are frequently used in portal frames, bridges, towers, and industrial structures because material can be distributed in accordance with the variation in internal force. This can reduce self-weight and improve structural efficiency. However, the stability response of a tapered member is more complex than that of a prismatic member because the flexural, torsional, and warping stiffness vary

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continuously along its length. General steel-design specifications provide broad stability requirements, but the direct application of prismatic-member expressions to strongly tapered members may not capture the influence of the changing section properties [1,2].

Early studies considered the elastic lateral buckling of non-uniform beams [3]. Later analytical and numerical investigations addressed the lateral-torsional buckling of tapered I-sections and showed that the response depends on the taper profile, moment distribution, end restraints, and cross-sectional symmetry [4,5]. These studies demonstrate that finite element analysis is an appropriate tool when closed-form solutions become difficult or are limited to idealized elastic cases. The objective of the present work is to examine the effect of the depth-taper ratio on the lateral-torsional buckling resistance of linearly tapered steel W-section members. The numerical procedure combines eigenvalue buckling analysis with an imperfect, geometrically nonlinear analysis. The study focuses on the variation in moment capacity for members having the same length and material properties but different taper ratios.

2. Geometry, Tapering Definition, and Scope

2.1 Member Geometry

The member depth varies linearly from D_0 at the larger end to D_1 at the smaller end, as illustrated in Figure 1. The available manuscript indicates that the web is tapered while the flange width and plate thickness remain constant along the member.#

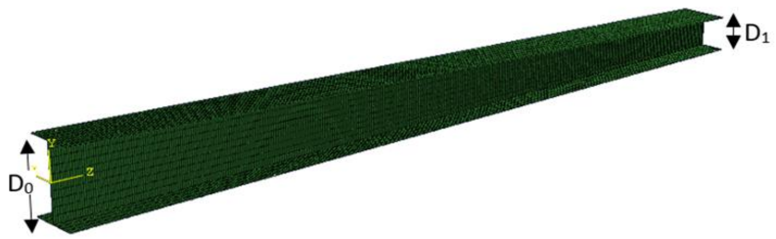

Figure 1: Geometry of linearly tapered w-section member.

2.2 Taper Ratio

Taper ratio describes the variation in section depth along its length, it is commonly defined as the ratio of the smaller section depth at one end to the larger section depth at the other end. To remain consistent with the investigated numerical range, the taper ratio is defined as:

$$\eta = D_1 / D_0 , \quad 0 < \eta \leq 1.0 \qquad (1)$$

A value of η = 1.0 represents a prismatic member, while smaller values represent increasing taper ratios.

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2.3 Scope and Assumptions

The members are straight, linearly tapered, and fabricated from steel.

The compression flange is laterally unbraced over the investigated length, permitting lateral-torsional buckling.

The end sections are constrained to remain plane, and warping restraint is prescribed through the end-boundary conditions.

The steel constitutive model is elastic-perfectly plastic.

Initial geometric imperfections are introduced using a scaled eigenvalue buckling mode.

3. Finite Element Modelling

3.1 Finite Element Analytical Modelling

Finite element analysis divides a structure into small parts that meet into nodes, stresses and strains can be easily simulated in each element. Finite element analysis is particularly useful for analysing tapered sections where mathematical calculations become complex. Finite element analysis allows engineers to visualize potential failure points and refine the design for maximum safety. Finite element models were developed using ABAQUS Version 6.14 (ABAQUS, 2014) to model the tapered cross sections. The models were evaluated under flexural loading assuming the compression flanges were unbraced and therefore, the models were intended to fail by lateral-torsional buckling. The moment capacities and the load-displacement results were obtained for different taper ratios.

Prior to performing a static stress analysis, initial geometric imperfections were implemented into the finite element models. The initial imperfections were a small percentage of the natural buckled shape from lateral-torsional buckling. The buckled shape was determined by performing an eigenvalue buckling analysis in ABAQUS with an example shown in Figure 2. In the model, the boundary conditions on the end ensured plane sections remain plane. For prismatic members and with no eccentricities in loading, buckling will not occur in a static stress analysis using finite elements. Therefore, the imperfection needed to be implemented in order for lateral-torsional buckling to occur in the column.

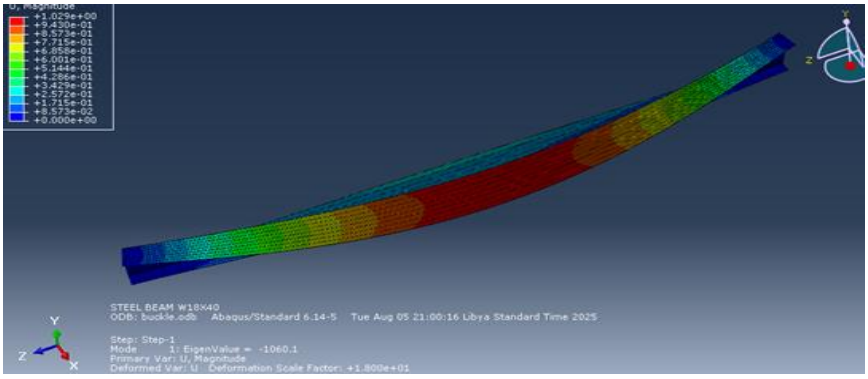

Figure 2: FEM Model Showing Buckled Shape

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3.2 Analytical Test Matrix

The analytical test matrices are presented in Tables 1&2 and identify the various parameters that were altered in the analytical investigations. Two different column sections have been studied with taper ratio in strong axis and subjected to constant bending moments at the ends. Sections at the ends are enforced to remain plane, and the steel is elastic-perfectly plastic.

Table 1: Geometric parameters of the finite element models for W8×18 section.

Model	D_0 (in.)	D_1 (in.)	t_w (in.)	t_f (in.)	b_f (in.)	L (in.)	$\eta = D_1/D_0$
Model 1	3.905	7.81	0.23	0.33	5.25	180	0.5
Model 2	5.077	7.81	0.23	0.33	5.25	180	0.65
Model 3	6.248	7.81	0.23	0.33	5.25	180	0.8
Model 4	7.81	7.81	0.23	0.33	5.25	180	1.0

3.3 Boundary Conditions and Loading

W-Sections were analyzed to evaluate the influence of tapering on the lateral torsional buckling capacity of steel columns. Pinned-pinned boundary conditions were considered, for simply supported boundary condition, the loading condition was uniform moment. The finite element section also states that equal end moments were applied to create flexural loading and lateral-torsional buckling

Table 2: Geometric parameters of the finite element models for W14×30 section.

Model	D_0 (in.)	D_1 (in.)	t_w (in.)	t_f (in.)	b_f (in.)	L (in.)	$\eta = D_1/D_0$
Model 1	6.90	13.8	0.27	0.385	6.73	180	0.5
Model 2	8.97	13.8	0.27	0.385	6.73	180	0.65
Model 3	11.04	13.8	0.27	0.385	6.73	180	0.8
Model 4	13.80	13.8	0.27	0.385	6.73	180	1.0

3.4 Material Properties

The values of the yield stress (σy), the ultimate stress, the yield strain (εy), the ultimate strain (εu), and the elastic modulus (E) are provided in Table 3. The material properties shown in table 3 were used in all models from the test matrices shown in Tables 1&2. Therefore, the W8X18 sections used the same material properties as the W14X30 sections. These material properties represent a real data derived from tension testing of W-sections.

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Table 3: Average Material Properties of Sections

σ_y (ksi)	σ_u (ksi)	ε_y (in/in)	ε_u (in/in)	E (ksi)
65	84.3	0.00171	0.0925	36433

3.5 Eigenvalue and Nonlinear Analyses

An eigenvalue buckling analysis was first performed for each tapered and prismatic model to obtain the governing lateral-torsional buckling mode. The selected eigenmode was then scaled and introduced into the initially perfect geometry as a geometric imperfection. The imperfect models were subsequently analyzed under equal end moments with geometric nonlinearity enabled. During the nonlinear analysis, the applied moment and the corresponding lateral displacement were recorded at each increment to generate the moment-displacement curves presented in Figures 3 and 4.

For all models, the numerical buckling moment was defined as the maximum moment reached on the moment-displacement curve, and the displacement at that same increment was taken as the displacement corresponding to buckling. This criterion is consistent with the computed response, which consists of an initial ascending branch, a distinct peak, and a descending post-peak branch. The descending branch represents a reduction in tangent stiffness following the onset of lateral-torsional instability. The same peak-extraction procedure was applied to the four taper ratios, η = 0.50, 0.65, 0.80, and 1.00, for both investigated W-Sections.

The eigenvalue analysis was used only to establish the imperfection shape, whereas the reported moment capacities were obtained from the imperfect geometrically nonlinear analyses. Because the peak and post-peak responses can be affected by the adopted imperfection amplitude, mesh density, load-increment size, and convergence controls, these modelling parameters should be reported explicitly and verified through sensitivity studies before the numerical capacities are used for design-oriented conclusions.

4. Results and Discussion

4.1 Moment-Displacement Response

The FEM analysis used for a deeper understanding of how changes in the taper ratio can impact the resistance of a column. Finite element analysis was used in order to obtain the initial imperfections and buckling shape. The buckling moment capacity for different taper ratios can easily be obtained from the analysis by FEM. The relation between the displacement and the moment was drawn for different taper ratios. The buckling moment capacity for different taper ratios can easily be obtained from the analysis by. The moment-displacement relationships indicate that elastic lateral torsional buckling occurred during the finite element analysis in all the cases.

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The finite element results of prismatic section were compared to the results of AISC equations to validate the models. The results compare well to each other, thus validating the models. Also, the results of tapered sections are compared with the results of prismatic sections. The comparisons indicate a change in the lateral torsional moment capacity of the tapered sections.

Figure 3 shows the results of W8×18 section. The Figure provides a comparison between the moment-displacement results of tapered column for different taper ratios. The Figure indicates change in the moment capacity of the section with different η when compared to non-tapered section. As the taper ratio increased from η = 0.5 to η =1.0 the buckling increased by about 12%. Reasons are related to the change in section properties. The same Figure shows a decrease in capacity ranges from (4%-8%) for η = 0.65 and η = 0.8 compared to the prismatic section.

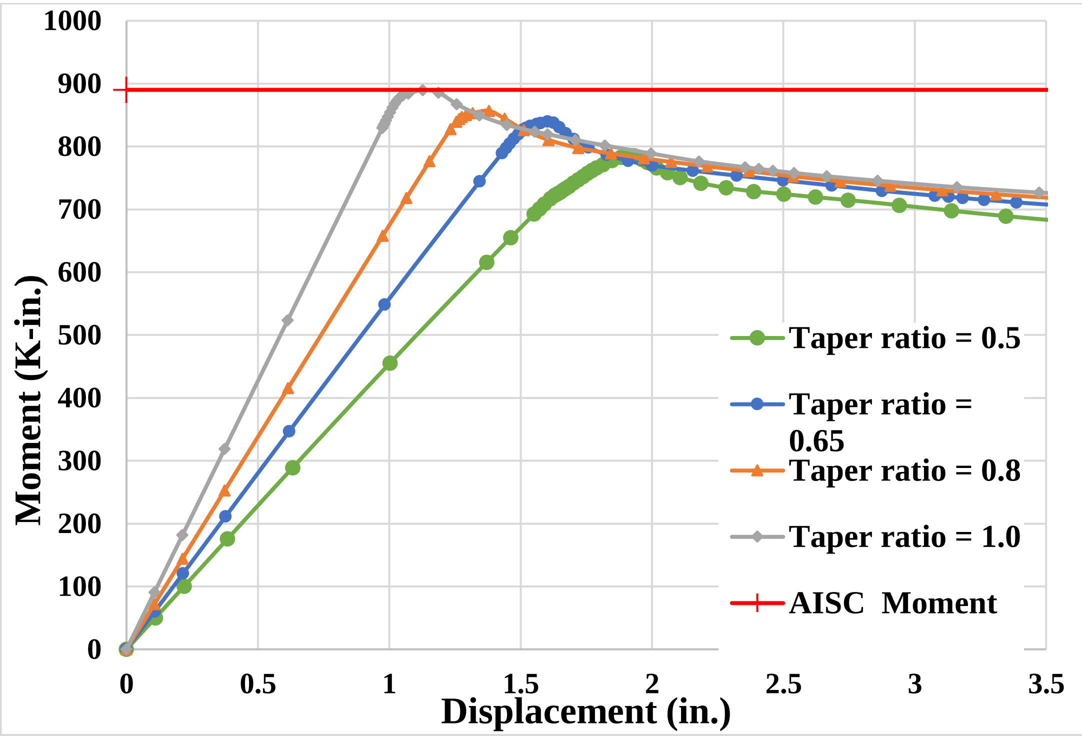

Figure 3: Moment-Displacement Results of W8×18 Section

Figure 4 shows the results of W14×30 section. The Figure shows a comparison between the relation between the displacement and the moment for different taper ratios, the figure shows that when the taper ratio decreases the buckling moment capacity decreases.

Buckling moment capacity increase linearly when the taper ratio changed from 0.5 to 1.0. The results show around 20% increasing in moment capacity between the two tapered ratios.

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The finite element analysis shows that the tapered steel column performs efficiently under the applied moment. The stress distribution is more uniform than that of a prismatic section because the cross area varies linearly. Maximum stresses are concentrate near the larger depth of the section, while minimum stresses located at the smaller end. The lateral displacement remains in acceptable range indicating adequate stiffness.

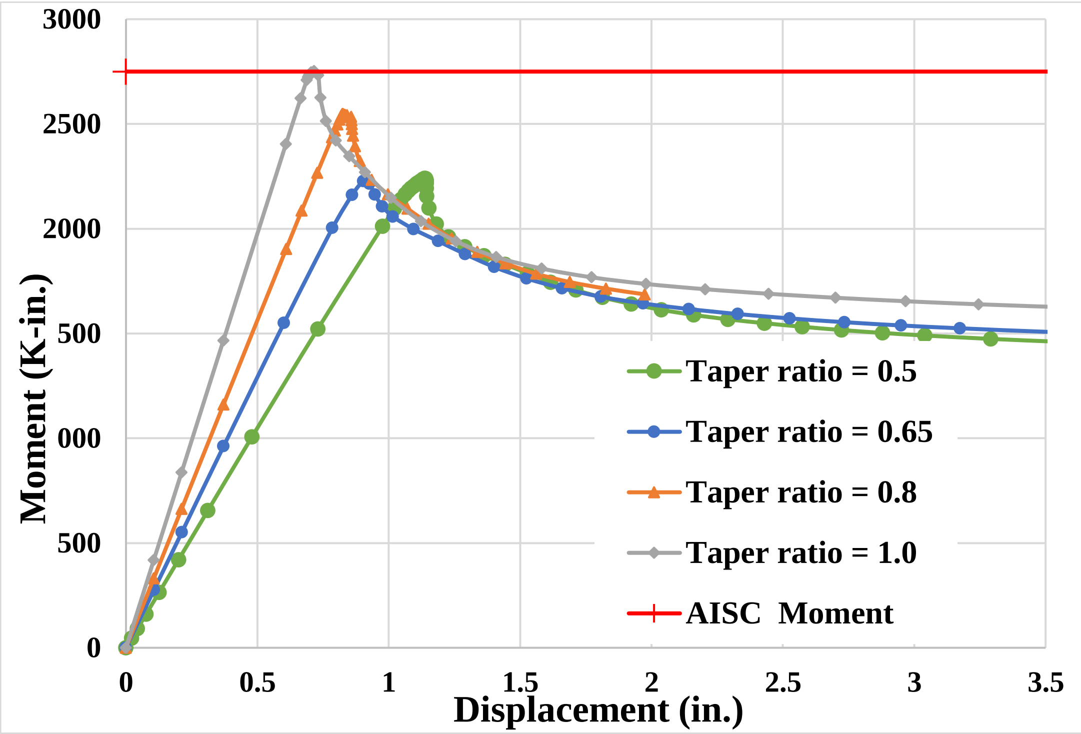

Figure 4: Moment-Displacement Results of W14×30 Section

4.2 Effect of Taper Ratio

The load displacement comparisons show that the taper ratio has a noticeable influence on the structural behavior of steel sections. This was determined by comparing load-displacement results of non-tapered and tapered sections using finite element models. The tapering has impact on the resistance was most pronounced for W14×30 members despite W8×18 members. Within the range η = 0.5-1.0, the original results report an approximately linear increase in moment capacity as η approaches 1.0. The limiting prismatic case is reported to have a capacity approximately 12% greater than the most strongly tapered case. This trend is physically plausible because increasing η increases the depth and stiffness over a larger portion of the member, but the magnitude of the increase cannot be independently assessed without the

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complete model data and numerical results. Table 4 shows a comparison between prismatic and tapered section for η = 0.5 & 1.0.

Table 4: comparison of taper ratio on section capacity.

Model	Peak moment (kip-in)	Displacement at peak (in.)	Reduction from η = 1.00 (%)
W8×18, η = 0.50	785.3	1.924	11.7
W8×18, η = 0.65	839.6	1.602	5.6
W8×18, η = 0.80	856.7	1.380	3.7
W8×18, η = 1.00	889.5	1.127	0.0
W14×30, η = 0.50	2241.2	1.138	18.6
W14×30, η = 0.65	2227.2	0.904	19.1
W14×30, η = 0.80	2547.2	0.825	7.5
W14×30, η = 1.00	2752.6	0.716	0.0

5. Conclusions

The current paper developed FEM analysis for tapered steel columns that can be used in stability analysis of steel frames with prismatic or linearly tapered columns. Tapered columns provide structural and economic advantages but require more advanced analysis for their stability. The buckling behaviour of tapered columns depend on taper ratio, end conditions, and material properties. The results indicate the following conclusions

- A two-stage finite element procedure consisting of eigenvalue buckling analysis followed by an imperfect geometrically nonlinear analysis can be used to investigate the lateral-torsional buckling of linearly tapered steel w-section members.
- For the investigated range η = 0.5-1.0, the original analysis reports an approximately linear increase in moment capacity as the member approaches the prismatic configuration.
- The reported difference in moment capacity between η = 0.5 and η = 1.0 is approximately 12% for the particular geometry, length, material model, loading, and end restraints considered.
- The findings should not be generalized to design practice until the model is fully documented and validated against an analytical, experimental, or independently published benchmark.

6. Limitations and Recommendations for Further Research

Based on the research that has been done, then there are some things that are still considered to be developed further to study the buckling behaviour of tapered columns including, boundary conditions, column length, and loading. Future work should examine

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additional taper ratios, member lengths, cross-sectional proportions, loading patterns, and end-restraint conditions. The influence of residual stresses, measured fabrication imperfections, material strain hardening, local plate buckling, and combined axial force and bending should also be investigated. Experimental testing or comparison with validated benchmark models is recommended before proposing design-oriented conclusions.

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